1) There is a set of modes that alternates with those identified with the cutoff condition of Schelkunoff in (4). Their cutoff condition is given by (5).

2) The modes identified with (4) have a cutoff that depends on ϵ, whereas the modes satisfying the alternate condition have a single cutoff value of p for all ϵ.

3) For $n>1$ all modes have some cutoff value. The principal mode for $n=1$ has no cutoff. There is no "degeneracy" of the modes for $n>1$ as there is for $n=1$ modes, as described by Beam [1]; that is, each mode has its own distinct cutoff point.

4) There is a unique principal mode for all n.

5) The existence of an additional mode between successive Schelkunoff modes reduces the upper frequency limit at which a pure principal mode may propagate below what the limit would be if only Schelkunoff modes existed, as follows.

The p–q curves may be used to determine the frequency dependence of the mode propagation with the aid of the additional relation

$$p^2 + q^2 = R^2 \tag{7}$$

where $R=(2a/\lambda_0)\pi\sqrt{\epsilon-1}$. This indicates that for a given dielectric rod of radius a, the actual values of p and q may be found at the intersection of the p–q curves with a superimposed circle of radius R corresponding to the frequency of operation for which the free-space wavelength is λ_0. To insure the propagation of a unique mode, the circle must intersect the p–q curves only once. For the principal mode, this means that the upper limit of R, and hence of frequency, is determined by the requirement that $R<p_0$, where $J_n(p_0)=0$. The lower limit of frequency is of course determined by the cutoff value of p (see Fig. 3).

It is now seen that there is no degeneracy to impose a notational distinction, so that the modes could be simply numbered successively. In the interest of conforming to the nomenclature for $n=1$, however, and in order to preserve the distinction between the modes that satisfy the Schelkunoff cutoff condition and those that satisfy the alternate condition, the HE_{nm}, EH_{nm} distinction is retained here, starting with HE_{n1} for the principal mode.

An attempt to verify a possible distinction between H- and E-type modes for the general case of any n, as suggested by Wegener and others [1], [7] for $n=1$, has not been found by the authors to lead to consistent results. The designation here of a mode as HE_{nm} is hence not to be construed as an indication that the mode must be H type.

The existence of the alternate cutoff condition, (5), has been confirmed independently using approximation methods by Snitzer [9] in the course of his investigation into the optical properties of thin fibers.

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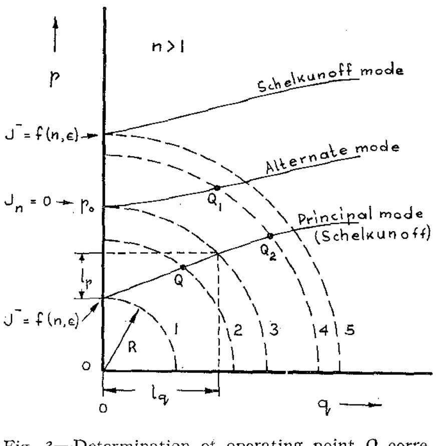

Fig. 3—Determination of operating point Q corresponding to a given frequency by superposition of a circle of radius R on p–q curves.

$$f(n,\epsilon) = \frac{1}{(n-1)(\epsilon+1)}, \qquad R = \frac{2a}{\lambda_0}\pi\sqrt{\epsilon-1}.$$

1) Lower limit of R. 2) Typical R. 3) Upper limit of R. 4) $R>p_0$. 5) Upper limit of R in absence of alternate modes. Q: operating point for typical R. Q_1, Q_2: two operating points for $R>p_0$: impure mode. l_p, l_q: useful ranges of p, q, for pure principal mode.

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A Property of Symmetric Hybrid Waveguide Junctions*

It is well known that in symmetric hybrid junctions such as the short-slot, branched-guide, and transvar types, the signals in the main and auxiliary guides are in phase quadrature. Another property of fully symmetric lossless hybrids is that if all the arms are matched, the amplitudes of the waves traveling in the reverse direction in the main and auxiliary guides are equal. A proof is given below.

Since in a well-designed hybrid these amplitudes will be of the order of 0.03 or less, relative to the input, this is not an easy fact to observe experimentally. However, if the measured VSWR and isolation of such a hybrid are inconsistent, one may reflect that this must be because of

1) experimental error,
2) mismatch of terminations or bends introduced for purposes of measurement,
3) asymmetry allowed by manufacturing tolerances,
4) ohmic loss.

Proof: Let arms 1–3 be the main guide, and arms 2–4 the auxiliary guide. If the hybrid is fully symmetric its scattering matrix will have the form,

$$S = \begin{pmatrix} A & B & C & D \\ B & A & D & C \\ C & D & A & B \\ D & C & B & A \end{pmatrix},$$

where A and B are small, and C and D have approximately equal amplitude. If the hybrid is lossless, S is unitary, which gives us

$$\text{Re}\,(A\bar{B}) + \text{Re}\,(C\bar{D}) = 0, \tag{1}$$
$$\text{Re}\,(A\bar{C}) + \text{Re}\,(B\bar{D}) = 0, \tag{2}$$
$$\text{Re}\,(A\bar{D}) + \text{Re}\,(B\bar{C}) = 0, \tag{3}$$

where the bar denotes the complex conjugate. Let $A=A_1+jA_2$, and similarly for B and D, and let the reference planes be chosen so that C is real. Then (1) shows that D_1 is a second-order small quantity, leading to the first property that C and D are in quadrature.

Putting $D=jC$, we have from (2) and (3), respectively,

$$A_1 = -B_2,$$
$$A_2 = -B_1.$$

Hence $A=-j\bar{B}$, and A and B have the same amplitude.

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Attenuation in a Resonant Ring Circuit*

The use of a resonant circuit will permit raising the strength of electromagnetic fields to values considerably higher than that available directly from a transmitter. In the usual resonant cavity, standing waves exist which may raise some doubt as to the usefulness of this method for testing certain

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